The differences operator*

Consider the problem of finding the k-th differences of a vector. Given a vector $\overrightarrow{V} = (v_1, v_2, v_3, v_4, \cdots, v_m)$ its 1st differences associated vector is defined as $\overrightarrow{W}$ an $(m-1)$ elements vector containing the differences:

$$w_i = v_{(i+1)} - v_i$$

Taking the 2nd differences for $\overrightarrow{V}$ is the same than taking the 1st differences for $\overrightarrow{W}$, this is, the first differences over the first differences. We proceed similarly for $k = 3$ (third differences), and any other higher order.

With some algebra it can be found out the following general result.

Viewed as definition for components:

$$vecDiff\,(v,k)_j = \left[(-1)^{(k \mod 2)}\right] \cdot \sum_{i=0}^{k} (-1)^{(i \mod 2)} \binom{k}{i} v_{j+i} \tag{1}$$

Viewed as an abstract vector operator:

$$vecDiff_{(k)} = |\mathbf{e_j} > \left[(-1)^{(k \mod 2)}\right] \sum_{i=0}^{k} (-1)^{(i \mod 2)} \binom{k}{i} < \mathbf{e^{j+i}}| \tag{2}$$

Notice that it makes sense to talk about k-th differences only when the vector has at least $(k+1)$ elements.

Also notice that the presented result is consistent with "zeroth order" differences $(k = 0)$ interpreted as the original vector itself, unchanged.

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